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ON THE LOCATION OF SPACECRAFT IN A GIVEN NUMBER OF ORBITS

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Space vehicles are an expensive product. For example, just putting such a device into orbit costs at least one hundred million dollars plus the cost of the satellite itself and scientific equipment it carries. However, the current state of human civilization does not allow us to do without the presence of satellites in orbit. There were 2,062 active satellites in the international database as of March 2019. Compared to 2018, the number of new devices increased by 15 %. Experts warn that in the coming years, the world is expecting a «satellite boom» with a projected increase in the number of devices of about 15–30 % annually. All these satellites are rather different. Currently, several orbits are used for placing satellites on them, depending on the tasks they solve. A geostationary orbit is used for live television broadcasting. Low satellite orbits are used for communication between satellite phones. There are some orbits for navigation systems (GPS, Navstar, GLONASS). Naturally, under these conditions, there is a problem of placing spacecraft over a given number of orbits, with some restrictions on the location of the spacecraft in certain orbits, depending on the purpose of the spacecraft. The solution to this problem is considered on the condition that the number of spacecraft coincides with the number of possible orbits in which they can be placed with some additional restrictions on the possibility of their placement in orbit. Several solutions to this problem are obtained that allow us to calculate the number of possible combinations for such placement of spacecraft over a given number of orbits.

Keywords: satellite, orbit, substitution, permanent.

О РАСПРЕДЕЛЕНИИ КОСМИЧЕСКИХ АППАРАТОВ ПО ЗАДАННОМУ ЧИСЛУ ОРБИТ

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Космические аппараты – дорогостоящий продукт. Например, только вывод такого аппарата на орбиту обходится минимум в сто миллионов долларов плюс стоимость самого спутника и научной аппаратуры, которую он несет. Однако современное состояние человеческой цивилизации уже не позволяет обходиться без наличия космических аппаратов на орбите. В международной базе данных на март 2019 г. числилось 2062 действующих спутника. По сравнению с 2018 г. рост числа новых аппаратов составил 15 %. Эксперты предупреждают, что в ближайшие годы мир ожидает «спутниковый бум» с прогнозируемым приростом количества аппаратов порядка 15–30 % ежегодно. Все эти космические аппараты сильно отличаются друг о друга. В настоящее время используется несколько орбит для размещения на них спутников в зависимости от решаемых ими задач. Геостационарная орбита используется для прямого телевещания. Низкие спутниковые орбиты используются для связи между спутниковыми телефонами. Свои орбиты существуют для спут-

ников систем навигации (GPS, Navstar, ГЛОНАСС), военных спутников, спутников для различных научных исследований. Естественно, в этих условиях возникает задача распределения космических аппаратов по заданному числу орбит при некоторых ограничениях на нахождении космического аппарата на некоторых орбитах в зависимости от назначения космического аппарата. Рассматривается решение данной задачи при условии, что число космических аппаратов совпадает с числом возможных орбит, на которых эти космические аппараты могут находиться при некоторых дополнительных ограничениях на возможность расположения спутника на орбите. Получено несколько решений этой задачи, позволяющих вычислить число возможных комбинаций для таких распределений космических аппаратов по заданному числу орбит.

Ключевые слова: спутник, орбита, подстановка, перманент.

1. Introduction. In the international database as for March 2019, there were 2062 active satellites. Compared to 2018, the growth of the number of new devices is 15 %. Experts warn that in the coming years the world is expecting a "satellite boom" with a projected increase of the number of devices of about 15–30 % annually. At present, some companies such as SpaceX and OneWeb are implementing an extensive program to launch satellites into low Earth orbits to put into action the plans for creating a global Internet broadcasting network.

In this case, a number of problems naturally arise [1–6], in particular, the problem of choosing the most favorable orbits for satellites of one class or another, as well as the problem of allocating a given number of satellites over a given set of orbits with existing prohibitions on placing a satellite in some orbit.

The article provides a solution to the second problem under the following conditions: the number of satellites *n* coincides with the number of possible orbits in which they are placed; each satellite is forbidden to be in exactly one orbit; two satellites cannot be in the same orbit. There is a formula that allows us to calculate the number of possible combinations for such satellite distributions in orbits.

The mathematical model of the problem. By permutation we mean the dimension matrix $2\times n$

$$\begin{pmatrix} 1 & 2\cdots & n \\ i_1 & i_2\cdots & i_n \end{pmatrix}. \quad (1)$$

A permutation is called regular if $k \neq i_k$, i. e.

$$i_1 \neq 1, i_2 \neq 2, \cdots, i_n \neq n. \quad (2)$$

Thus, in accordance with the restrictions indicated above, exactly one regular permutation of degree *n* will correspond to each option for the placement of satellites in orbits. The problem of finding the number of regular permutations of degree *n* belongs to an extensive class of problems of enumerating permutations with restrictions on positions (with forbidden positions), and it is referred to as the problem of derangements (le problem des recontres); [7]).

It is equivalent to the classical problem of enumerating Latin rectangles having the size $2\times n$; for it, as the problem of combinatorics, many solutions of various types are known ([8], chapters 7, 8). The article presents several such solutions made using various methods, each of which has an independent value.

2. Recurrence formula for calculating the number of regular permutations. The number of regular permutations of degree *n* will be denoted by D_n.

Theorem 1. If $n=1$, then $D_1=0$. If $n=2$, then $D_2=1$. If $n>2$, then

$$D_n = n! - C_n^1 D_{(n-1)} - C_n^2 D_{(n-2)} - \cdots - C_n^{(n-2)} D_{n-(n-2)} - 1.$$

Proof. The first two conclusions of the theorem are a direct consequence of the definition of a regular permutation. Let $n>2$. Let us consider the irregular permutation

$$\begin{pmatrix} 1\cdots n_1 \cdots n_k \cdots & n \\ \alpha_1 \cdots n_1 \cdots n_k \cdots & \alpha_n \end{pmatrix},$$

which holds fixed precisely *k* symbols of $n_1, n_2, \cdots, n_k$, and it moves all the remaining symbols (in quantity $k-1$). At fixed $n_1, n_2, \cdots, n_k$ the number of such permutations is D_{n-k}. Since the number of different sets without reiteration of the length *k* from the range $\{1,\cdots,n\}$ equals C_n^k, then the total number of irregular permutations of this kind equals C_n^k. To complete the proof of the theorem, it is enough to subtract all irregular permutations of the form considered above for all *k* from the total number of permutations of the degree *n* (and this is exactly $n!$).

The theorem is proved.

Let us verify the conclusions of the theorem for some initial values $n = 1, 2, 3, 4, 5, 6$. The values $D_1 = 0$ and $D_2 = 1$ follow directly from the definition

$$D_3 = 3! - C_3^1 D_2 - 1 = 6 - 3 - 1 = 2,$$

$$D_4 = 4! - C_4^1 D_3 - C_4^2 D_2 - 1 = 24 - 4\cdot 2 - 6\cdot 1 - 1 = 9,$$

$$D_5 = 5! - C_5^1 D_4 - C_5^2 D_3 - C_5^3 D_2 - 1 =$$
$$= 120 - 5\cdot 9 - 10\cdot 2 - 10 - 1 = 44,$$

$$D_6 = 6! - C_6^1 D_5 - C_6^2 D_4 - C_6^3 D_3 - C_6^4 D_2 - 1 =$$
$$= 720 - 6\cdot 44 - 15\cdot 9 - 20\cdot 2 - 15\cdot 1 - 1 = 265.$$

The results have been confirmed by listing all the regular permutations for the specified values $n = 1, 2, 3, 4, 5, 6$.

3. The formulae for calculating the number of regular permutations based on the concept of a permanent matrix. Here we consider the original task as an enumeration problem when calculating the number of the permutations with fixed positions and we use the means of permanents (cyclic) (0,1) of incidence matrices [9; 10]. By the permanent of a square $n\times n$ matrix $A=(a_{i,j})$ over a commutative ring we mean the expression

$$per(A) = \sum_{\sigma\in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}, \quad (3)$$

where summation is carried out for all S_n permutations $\sigma = (\sigma(1), \cdots, \sigma(n))$ n-of the range $\{1, 2, \cdots, n\}$, as well as on all the diagonals of the matrix A.

Let I_n be identity $n \times n$ matrix, J_n – $n \times n$ -the matrix, in which each element equals $1/n$, and $A_n = (nJ_n - I_n)$ – the following matrix of order n:

$$A_n = \begin{pmatrix} 0 & 1 & \cdots & 1 & 1 \\ 1 & 0 & & 1 & 1 \\ \vdots & & \ddots & & \vdots \\ 1 & 1 & & 0 & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix} \quad (4)$$

Theorem 2. The following formula for the number D_n is valid

$$D_n = per(A_n) = per(nJ_n - I_n), n = 2, 3, \cdots. \quad (5)$$

Proof. The incidence matrix $A_n = (a_{ij})$ of the order n in (4) is constructed in the way that in accordance with the conditions (2) it has $a_{ij} = 0$, if $i = j$, and $a_{ij} = 1$, if $i \neq j$. We can easily see that for each "identity" diagonal of the matrix A_n in (4), each element of which equals 1, the permutation of the type (2) corresponds one-to-one. On the other hand, it is obvious that the multiplication of n units equal to *1* corresponds to each such diagonal in the permanent sum (3) for $per(A_n)$. At the same time, the corresponding multiplications of the elements for each diagonal of the matrix A_n containing at least one zero element equal zero.

The theorem is proved.

Theorem 3. The following formula for the number D_n is valid

$$D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + \left((-1)^n \frac{1}{n!}\right), n = 2, 3, \cdots. \quad (6)$$

Proof. By the "permanent" Laplace formula, decomposing $per(A_n)$ into the elements of the first line, we have

$$D_n = per(A_n) = \sum_{k=2}^{n} per\left(A_n\left(1/k\right)\right), n = 3, 4, \cdots. \quad (7)$$

Here $per\left(A_n\left(1/k\right)\right)(k = 2, 3, \cdots, n)$ – permanental minors from (0,1) matrices $A_n\left(1/k\right)$ of the order $(n-1)$, which differ only in the arrangement of the same (0,1)-lines.

Herewith each matrix $A_n\left(1/k\right)$ contains precisely $(n-2)$ zeros, and no two of them are contained in one line or in one column. It follows that at any $k = 2, 3, \cdots, n$ the following equation is valid

$$per\left(A_n\left(1/k\right)\right) = per(A_{n-1}) - per(A_{n-2}),$$

and according to (7) and (5) we obtain

$$\begin{aligned} D_n &= (n-1)\left(per(A_{n-1}) - per(A_{n-2})\right) = \\ &= (n-1)(D_{n-1} + D_{n-2}), n = 3, 4, \cdots, n. \end{aligned} \quad (8)$$

Therefore, by induction, taking into account the initial conditions $D_1 = 0, D_2 = 1,$ the formula (6) follows.

From here it follows that

The theorem is proved.

4. Derivation of the formula for calculating the number of regular permutations based on the inclusion-exclusion formula. This solution is related to the classical inclusion-exclusion formula (please see, for example, [10], § 3.2) by breaking the set S_n into disjoint sets $P_0, P_1, \cdots, P_n$, where P_i – set of permutations from S_n with precisely i fixed positions, $i = 1, 2, \cdots, n$ or, the same with $(n-i)$ constraint positions. Since

$$S_n = n! = \sum_{i=0}^{n} |P_i|, |P_i| = \binom{n}{i} D_{n-i},$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}, \ i = 1, 2, \cdots, n,$$

We have

$$1 = \sum_{i=o}^{n} D_{n-i} \frac{1}{i!(n-i)!}. \quad (9)$$

Let

$$\tilde{D}_z = \sum_{i=0}^{\infty} \frac{D_i}{i!}$$

– generating function of a power (exponential) type for a numerical sequence $\left\{D_i / i!\right\}$, and consequently,

$$D_i = i! rez \left\{ \tilde{D}(z) z^{-i-1} \right\}, i = 1, 2, \cdots, n \quad (10)$$

Here by formal deduction operator $res_z \{A(z)\}$ for the power Laurent series

$$A(z) = \sum_{k=r}^{\infty} a_k z^k, r \in Z,$$

we mean the coefficient at z^{-1} of the range $A(z)$, i. e., by convention,

$$res_z \{A(z)\} = a_{-1.} \quad (11)$$

Further, according to the coefficient method [11], we have in (9)

$$\frac{1}{i!} = res_w \left\{ e^w w^{-i-1} \right\}, \frac{D_{n-i}}{(n-1)!} = res_z \left\{ \tilde{D}(z) z^{-(n-k)-1} \right\},$$

and we sequentially obtain

$$1 = \sum_{i=0}^{n} \left(res_w \left\{ e^w w^{-i-1} \right\} \times res_z \left\{ \tilde{D}(z) z^{-(n-k)-1} \right\} \right) = \sum_{i=0}^{n} \cdots,$$

where in the last equation the added elements of the sum equal 0 by the definition of the operator res_z. Further, by regrouping the terms of the sum and putting the operator Σ under the symbol of the operator res_z, we obtain

$$1 = res_z\{\tilde{D}(z)z^{-n-1}[\sum_{i=0}^{\infty} z^w\left(res_w\left\{e^w w^{-i-1}\right\}\right)]\} =$$

(summation in square brackets by index i: substitution rule, substituting $w = z$)

$$= res_z\{\tilde{D}(z)z^{-n-1}[e^w]_{w=z}\} = res_z\left\{e^z\tilde{D}(z)z^{-n-1}\right\}.$$

That is, we have

$$res_z\left\{e^z\tilde{D}(z)z^{-n-1}\right\} = 1 \rightarrow := res_z\left\{\frac{1}{1-z}z^{-n-1}\right\}, n = 0,1,\cdots$$

It immediately follows that

$$e^z\tilde{D}(z) = \frac{1}{1-z},\ \tilde{D}(z) = \frac{e^{-1}}{1-z},$$

and according to (10)

$$D_n = n!res_z\left\{\tilde{D}(z)z^{-n-1}\right\} = n!res_z\left\{e^z\frac{1}{1-z}z^{-n-1}\right\} =$$

$$= n!res_z\left\{\left(1+\sum_{i=0}^{\infty}(-1)^i\frac{z^i}{i!}\right)\times\left(1+\sum_{i=0}^{\infty}(-1)^i z^i\right)z^{-n-1}\right\} =$$

(according to the operator res_z)

$$= n!(1-\frac{1}{1!}+\frac{1}{2!}-\cdots+\left(-1\right)^n\frac{1}{n!}), n = 1,2,3,\cdots.$$

5. Derivation of a formula for calculating the number of regular permutations based on combinatorial methods. This solution is connected to the use of traditional means of combinatorial analysis and generating functions of a power type related to the use of operations on power series (please see, for example [8–12]). Let Q_n – the set of all n-permutations of type (2), and $Q_n(k)$ – the set of these permutations from Q_n that have $1 \to k(k = 2,3,\cdots,n)$. Obviously, the class of sets $Q_n(k)$ – the partition of the set S_n. Let

$$Q_n(k) = Q_n^1(k) \cup Q_n^2(k),$$

where the set $Q_n^1(k)$ contains all the permutations from $Q_n(k)$ whose k does not image into 1, and the set $Q_n^2(k)$ contains all the permutations from $Q_n(k)$ whose k images into 1.

It stands to reason that for each fixed k we have

$$\left|Q_n^1(k)\right| = D_{n-1}, \left|Q_n^2(k)\right| = D_{n-2},$$

and consequently,

$$Q_n(k) = D_{n-1} + D_{n-2}.$$

Thus, we have

$$D_n\left|Q_n\right| = \sum_{k=0}^{n} Q_n(k) = \sum_{k=0}^{n}(D_{n-1}+D_{n-2}), n = 3,4, \quad (12)$$

i. e. the recurrence formula (8) taking into account the initial conditions of the form

$$D_1 = 0,\ D_2 = 1, \quad (13)$$

at $n \geq 3$ is valid.

System solution (12)–(13). If we divide both sides of the equality (12) by $(n-1)!$, multiply by z^{n-1}, sum up at $n \geq 3$, then taking into account the notation

$$\tilde{D}_n = D_n/n!,\ n \geq 1$$

we obtain the following equation

$$\sum_{n=3}^{\infty}\frac{D_n}{(n-1)!}z^{n-1} = z\sum_{n=3}^{\infty}\frac{D_{n-1}}{(n-2)!}z^{n-1} + z\sum_{n=3}^{\infty}\frac{D_{n-2}}{(n-2)!}z^{n-2}. \quad (14)$$

Further, from (14), taking into account the initial conditions (13) and the new notation

$$\tilde{D}_n := D_n/n!, n \geq 2, \tilde{D}(z) := \sum_{n=1}^{\infty} z^n\frac{\tilde{D}_n}{n!} = \sum_{n=2}^{\infty} z^n,$$

we obtain

$$\sum_{n=3}^{\infty}\frac{D_n}{(n-1)!}z^{n-1} - z\sum_{n=3}^{\infty}\frac{D_{n-1}}{(n-2)!}z^{n-2} =$$

$$= -z^2D_2 + (1-z)\sum_{n=3}^{\infty}\frac{D_n}{(n-1)!}z^{n-1} =$$

$$= -z^2 + (1-z)\frac{d}{dz}\{\sum_{n=3}^{\infty}\frac{D_n}{n!}z^n\} =$$

$$= -z^2 + (1-z)\frac{d}{dz}\left\{\tilde{D}(z) - D_2z^2\right\} = \quad (15)$$

$$= -z^2 + (1-z)\frac{d}{dz}\left\{\tilde{D}(z)\right\} - (1-z)\frac{1}{2}2z =$$

$$= -z^2 + (1-z)\frac{d}{dz}\left\{\tilde{D}(z) - D_2z^2\right\} =$$

$$= -z + (1-z)\frac{d}{dz}\left\{\tilde{D}(z)\right\}.$$

It follows from here that

$$z\sum_{n=3}^{\infty}\frac{D_{n-2}}{(n-2)!}z^{n-2} = z\sum_{n=3}^{\infty}\tilde{D}_{n-2}\,z^{n-2} =$$

$$= z\sum_{n=3}^{\infty}\tilde{D}_{n-2}\,z^{n-2} = z\left\{\tilde{D}(z) - z\tilde{D}_1\,z\right\} = z\tilde{D}(z). \quad (16)$$

Thus, the solution to the system (12) – (13) by virtue of (14) – (16) reduces to solving the following first order differential equation:

$$(1-z)\frac{d}{dz}\left\{\tilde{D}(z)\right\} - z\tilde{D}(z) - z = 0, \tilde{D}(0) = \tilde{D}'(0) = 0. \quad (17)$$

The equation (17) has the following unambiguous solution

$$\tilde{D}(z) = -1 + e^{-z}\frac{1}{1-z}, \quad (18)$$

which is verified by direct substitution in (17). The end of the proof is carried out exactly as in Section 4.

6. Notices. From the Maclaurin decomposition for the number e^{-1} the known next important estimate of the number D_n follows immediately:

$$\lim_{n\to\infty}\frac{D_n}{n!} = \frac{1}{e}. \quad (19)$$

In 1979, M. Aigner found the following unexpected probabilistic interpretation of that asymptotic estimate of the number D_n : «Let us suppose that n pages of a manuscript were mixed up by a gust of wind and then put arbitrarily.

The formula (19) states that for bigger n, the probability that there is not a single page in its place is higher than $1/3$. » [13].

Another, more difficult classical combinatorial problem of the same type is the following: The problem of married couples: in how many ways can couples be arranged around a round table so that women and men alternate, and none of wives is sitting next to her husband?

In 1934, by listing enumerated n-permutations of the corresponding type, Jacques Toucheard found the following formula for the number U_n – ways to place married couples of the above type [14; 15]:

$$U_n = per\left(nJ_n - I_n - P_n\right) = \\ = \sum_{i=0}^{n} -1^i \frac{2n}{2n-1}\binom{n-i}{i}\left(n-1\right)!, \quad (20)$$

where the matrix P_n – permutation matrix of the degree n with 1 on the places $(1,2),(2,3),\cdots,...$

Let us note that the formula (20), as well as the formula (6), can also be used to calculate options for the placement of satellites on a given number of orbits with a different set of conditions.

Conclusion. The problem of the possible distribution of satellites over a given number of orbits, depending on the purpose of a satellite, is solved. It is assumed that the number of satellites coincides with the number of orbits on which they can be placed. Several solutions to this problem have been obtained, which make it possible to calculate the number of combinations for distributing satellites over a given number of orbits.

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